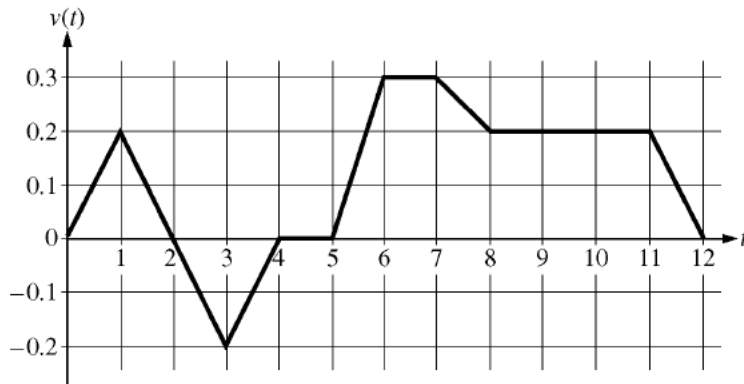


Name:  
 Teacher:  
 Period:  
 Due Date:

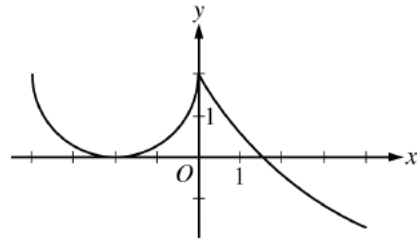


1. Caren rides her bicycle along a straight road from home to school, starting at home at time  $t = 0$  minutes and arriving at school at time  $t = 12$  minutes. During the time interval  $0 \leq t \leq 12$  minutes, her velocity  $v(t)$ , in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.
  - (a) Find the acceleration of Caren's bicycle at time  $t = 7.5$  minutes. Indicate units of measure.
  - (b) Using correct units, explain the meaning of  $\int_0^{12} |v(t)| dt$  in terms of Caren's trip. Find the value of  $\int_0^{12} |v(t)| dt$ .
  - (c) Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.
  - (d) Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function  $w$  given by  $w(t) = \frac{\pi}{15} \sin\left(\frac{\pi}{12}t\right)$ , where  $w(t)$  is in miles per minute for  $0 \leq t \leq 12$  minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

|        |   |   |    |   |    |
|--------|---|---|----|---|----|
| $x$    | 2 | 3 | 5  | 8 | 13 |
| $f(x)$ | 1 | 4 | -2 | 3 | 6  |

5. Let  $f$  be a function that is twice differentiable for all real numbers. The table above gives values of  $f$  for selected points in the closed interval  $2 \leq x \leq 13$ .
  - (a) Estimate  $f'(4)$ . Show the work that leads to your answer.
  - (b) Evaluate  $\int_2^{13} (3 - 5f'(x)) dx$ . Show the work that leads to your answer.
  - (c) Use a left Riemann sum with subintervals indicated by the data in the table to approximate  $\int_2^{13} f(x) dx$ . Show the work that leads to your answer.
  - (d) Suppose  $f'(5) = 3$  and  $f''(x) < 0$  for all  $x$  in the closed interval  $5 \leq x \leq 8$ . Use the line tangent to the graph of  $f$  at  $x = 5$  to show that  $f(7) \leq 4$ . Use the secant line for the graph of  $f$  on  $5 \leq x \leq 8$  to show that  $f(7) \geq \frac{4}{3}$ .

Name:  
 Teacher:  
 Period:  
 Due Date:

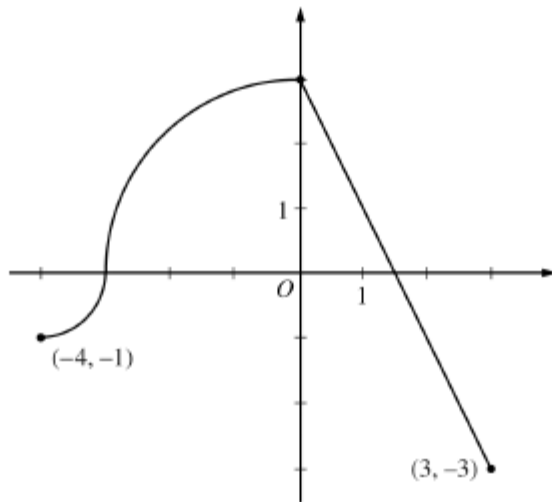


Graph of  $f'$

6. The derivative of a function  $f$  is defined by  $f'(x) = \begin{cases} g(x) & \text{for } -4 \leq x \leq 0 \\ 5e^{-x/3} - 3 & \text{for } 0 < x \leq 4 \end{cases}$

The graph of the continuous function  $f'$ , shown in the figure above, has  $x$ -intercepts at  $x = -2$  and  $x = 3 \ln\left(\frac{5}{3}\right)$ . The graph of  $g$  on  $-4 \leq x \leq 0$  is a semicircle, and  $f(0) = 5$ .

- (a) For  $-4 < x < 4$ , find all values of  $x$  at which the graph of  $f$  has a point of inflection. Justify your answer.
- (b) Find  $f(-4)$  and  $f(4)$ .
- (c) For  $-4 \leq x \leq 4$ , find the value of  $x$  at which  $f$  has an absolute maximum. Justify your answer.



Graph of  $f$

4. The continuous function  $f$  is defined on the interval  $-4 \leq x \leq 3$ . The graph of  $f$  consists of two quarter circles and one line segment, as shown in the figure above. Let  $g(x) = 2x + \int_0^x f(t) dt$ .

- (a) Find  $g(-3)$ . Find  $g'(x)$  and evaluate  $g'(-3)$ .
- (b) Determine the  $x$ -coordinate of the point at which  $g$  has an absolute maximum on the interval  $-4 \leq x \leq 3$ . Justify your answer.
- (c) Find all values of  $x$  on the interval  $-4 < x < 3$  for which the graph of  $g$  has a point of inflection. Give a reason for your answer.
- (d) Find the average rate of change of  $f$  on the interval  $-4 \leq x \leq 3$ . There is no point  $c$ ,  $-4 < c < 3$ , for which  $f'(c)$  is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.