Name:
Teacher:
Period:
Due Date:

Be Good
Why>How>What

HW 4.3

So far we have only been using the power rule to determine antiderivatives. Initially while learning this, we noted that $\frac{d}{dx}[x^n] = nx^{n-1}$. We used that rule to determine that $\int x^n dx = \frac{1}{n+1}x^{n+1} + C$. We're now going to be able to reverse the Chain Rule for differentiation. Indeed, though integration has a product rule, this falls outside the scope of the first course of Calculus. That said, the Chain Rule for integration is the last major rule we will learn. And it cannot be stressed, any antiderivative that cannot be found without the Power Rule or Chain Rule, will be unable to be done this year (and may be impossible to do at all).

1.) Differentiate where u is a function of x.

$$\frac{d}{dx}[f(u)]$$

2.) Integrate

$$\int f'(u)du$$

3.) Differentiate.

$$\frac{d}{dx}[(x^2+1)^3]$$

4.) Integrate

$$\int (x^2+1)(2x)dx$$

5.) Exemplar to recognize $f'(u)du$ pattern.

$$\int \cos x\,(1+\sin x)^2 dx$$

$$\int \underbrace{\cos x}_{u'} \underbrace{(\overbrace{1+\sin x}^{u})^2}_{f'} dx$$

Annotated integral

$$f'(u) = u^2$$

$f'(u)$ in terms of u

$$u = 1+\sin x$$

u in terms of x

$$du = \cos x\, dx$$

Determine and solve for du

$$\int u^2 du = \frac{1}{3}u^3 + C$$

Integral rewritten as $f'(u)du$

$$= \frac{1}{3}(1+\sin x)^3 + C$$

Anti-derivative in terms of x with $+C$

Criteria for Success

A successful antiderivative will…

- Contain an annotated integral
- Explicitly identify $f'(u)$ in terms of u
- Explicitly identify u in terms of x
- Determine and solve for du
- Rewrite integral as $f'(u)du$
- Write the final anti-derivative in terms of x
- Antiderivative is written without rational or negative exponents.

6.) Integrate

$$\int (8x^2+1)^2(16x)dx$$

$$\int \frac{2x}{\sqrt{x^2+1}}dx$$

$$\int 3x^2\sqrt{x^3+5}dx$$

Name:
Teacher:
Period:
Due Date:

Be Good
Why>How>What

HW 4.3

- Antiderivative contains $+C$

7.) Exemplar that requires a multiplication by a constant.

$$\int \frac{x}{(x^2+4)^2}\,dx$$

$$\int \underbrace{\frac{x}{(x^2+4)^2}}_{f'(u)}\,dx$$

Annotated integral

$$f'(u) = \frac{1}{u^2} = u^{-2}$$

$f'(u)$ in terms of u

$$u = x^2+4$$

u in terms of x

$$du = 2x\,dx$$

$$\frac{1}{2}du = x\,dx$$

Determine and solve du equation for what is present in integral.

$$\int \frac{1}{2}u^{-2}\,du = -\frac{1}{2}u^{-1} + C$$

Integral rewritten as $f'(u)du$

$$= -\frac{1}{2}(x^2+4)^{-1} + C = \frac{-1}{2(x^2+4)} + C$$

Anti-derivative in terms of x with $+C$

Criteria for Success

A successful antiderivative will…

- Contain an annotated integral
- Explicitly identify $f'(u)$ in terms of u
- Explicitly identify u in terms of x
- Determine and solve du equation for what is present in integral.
- Rewrite integral as $f'(u)du$
- Write the final anti-derivative in terms of x
- Antiderivative is written without rational or negative exponents.
- Antiderivative contains $+C$

8.) Integrate

$$\int (8x^2+1)^2(16x)dx$$

$$\int \frac{2x}{\sqrt{x^2+1}}dx$$

$$\int 3x^2\sqrt{x^3+5}dx$$

Name:
Teacher:
Period:
Due Date:

Be Good
Why>How>What

HW 4.3

Two unannotated problems with solutions are now presented. The problem on the left is another like the last section, the problem on the right what we call a “Change of Variables” problem. For the two…

- Compare and contrast the methods of solutions.
- Explain what substitution has taken place in the circled passages.
- Generalize the solution on the right to solve the practice problems.

$$\int \sqrt{2x-1}dx$$

$$\int \underbrace{\sqrt{2x-1}}_{f'(u)}\,dx$$

$u = 2x-1 \qquad f'(u) = u^{1/2}$

$du = 2dx$

$\frac{1}{2}du = dx$

$$\int \frac{1}{2}u^{1/2}du$$

$= \frac{1}{2}\cdot\frac{u^{3/2}}{3/2} + C$

$= \frac{1}{3}\sqrt{(2x-1)^3} + C$

$$\int x\sqrt{2x-1}dx$$

$$\int x\underbrace{\sqrt{2x-1}}_{f'(u)}\,dx$$

$u = 2x-1 \qquad f'(u) = u^{1/2}$

$x = \frac{u+1}{2}$

$du = 2dx$

$dx = \frac{1}{2}du$

$$\int \left(\frac{u+1}{2}\right)u^{1/2}\left(\frac{1}{2}du\right)$$

$$\int \frac{1}{4}\left(u^{3/2} + u^{1/2}\right)du$$

$\frac{1}{4}\int u^{3/2}du + \frac{1}{4}\int u^{1/2}du$

$\frac{1}{4}\cdot\frac{u^{5/2}}{5/2} + \frac{1}{4}\cdot\frac{u^{3/2}}{3/2} + C$

$\frac{1}{10}\sqrt{(2x-1)^5} + \frac{1}{6}\sqrt{(2x-1)^3} + C$

9.) Integrate

$$\int \frac{x}{\sqrt{4x+5}}dx$$

10.) This cell is blank! How fun!

Name:
Teacher:
Period:
Due Date:

Be Good
Why>How>What

HW 4.3

11. $\int (1 + 6x)^4(6)\,dx$

12. $\int (x^2 - 9)^3(2x)\,dx$

13. $\int \sqrt{25 - x^2}\,(-2x)\,dx$

14. $\int \sqrt[3]{3 - 4x^2}(-8x)\,dx$

15. $\int x^3(x^4 + 3)^2\,dx$

16. $\int x^2(x^3 + 5)^4\,dx$

17. $\int x^2(x^3 - 1)^4\,dx$

18. $\int x(5x^2 + 4)^3\,dx$

19. $\int t\sqrt{t^2 + 2}\,dt$

20. $\int t^3\sqrt{t^4 + 5}\,dt$

21. $\int 5x\,\sqrt[3]{1 - x^2}\,dx$

22. $\int u^2\sqrt{u^3 + 2}\,du$

23. $\int \frac{x}{(1 - x^2)^3}\,dx$

24. $\int \frac{x^3}{(1 + x^4)^2}\,dx$

25. $\int \frac{x^2}{(1 + x^3)^2}\,dx$

26. $\int \frac{x^2}{(16 - x^3)^2}\,dx$

27. $\int \frac{x}{\sqrt{1 - x^2}}\,dx$

28. $\int \frac{x^3}{\sqrt{1 + x^4}}\,dx$

29. $\int \left(1 + \frac{1}{t}\right)^3\left(\frac{1}{t^2}\right)dt$

30 $\int \left[x^2 + \frac{1}{(3x)^2}\right]dx$

67. $\int x\sqrt{x + 6}\,dx$

68. $\int x\sqrt{4x + 1}\,dx$

69. $\int x^2\sqrt{1 - x}\,dx$

70. $\int (x + 1)\sqrt{2 - x}\,dx$

71. $\int \frac{x^2 - 1}{\sqrt{2x - 1}}\,dx$

72. $\int \frac{2x + 1}{\sqrt{x + 4}}\,dx$

73. $\int \frac{-x}{(x + 1) - \sqrt{x + 1}}\,dx$

74. $\int t\,\sqrt[3]{t + 10}\,dt$