Name:
Teacher:
Period:
Due Date:

Be Good
Why>How>What

HW 2.2A

The Chain Rule

If $y = f(u)$ is a differentiable function of u and $u = g(x)$ is a differentiable function of x, then $y = f\big(g(x)\big)$ is a differentiable function and

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Or, equivalently,

$$\frac{d}{dx}[f(u)] = f'(u) \cdot u'$$

Example

Differentiate:

$$\frac{d}{dx}\left[\frac{1}{\sqrt{4x^2 - x}}\right]$$

Our first job is to identify a function u in terms of x such that $f(u) = \frac{1}{\sqrt{4x^2-x}}$. In this case, we will set $u = 4x^2 - x$ and $f(u) = \frac{1}{\sqrt{u}}$. In this case, the inner function, u can be easily differentiated: $u' = 8x - 1$

$$\frac{d}{dx}\left[\frac{1}{\sqrt{u}}\right] = \frac{d}{dx}\left[u^{-1/2}\right]$$

By the chain rule:

$$\frac{d}{dx}\left[u^{-1/2}\right] = -\frac{1}{2}u^{-3/2} \cdot u'$$

$$= (8x - 1) \cdot -\frac{1}{2} \cdot (4x^2 - x)^{-3/2}$$

$$= -\frac{8x - 1}{2\sqrt{(4x^2 - x)^3}} = \frac{1 - 8x}{2\sqrt{(4x^2 - x)^3}}$$

Practice

Example A: Determine $f'(x)$ given $f(x) = (5x - 8)^4$

Example B: Determine $f'(x)$ given $f(x) = \sqrt{x^3 - 6}$

Answers

Example A: $f'(x) = 20(5x - 8)^3$

Example B: $f'(x) = \frac{3x^2}{2\sqrt{x^3-6}}$