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Be Good
Why>How>What

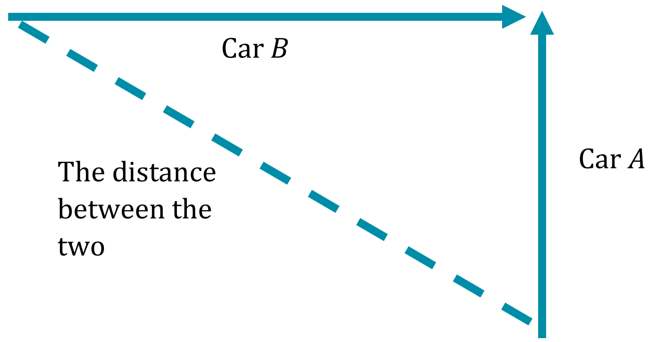
Gateway R Practice

Related Rates

We know from implicit differentiation, that when taking the derivative of an implied function of x, we must use chain rule, e.g.

$$\frac{d}{dx}[xy^2] = y^2 + 2xy\frac{dy}{dx}$$

This rule holds true when we are differentiating with multiple variables that are implied functions. We'll review a common math problem of two cars, one driving North toward an intersection and one driving East toward the same intersection. A common If Car A, the one driving north, at 35 miles per hour, is 9 miles away and the car driving East at 25 miles per hour, Car B is 12 miles away, a common pre-Calculus question might be, "How far away from each other are the two cars?" We can draw a diagram of this:



Application of our old friend $a^2 + b^2 = c^2$ will tell us that the cars are 15 miles apart. Not too hard. Calculus is the study of change, so to change this to a Calculus context, we can ask, "At what rate is the distance between the two cars *chang*ing?" When talking rate, this means we need to take the derivative, but each of the three variables in the equation from the Pythagorean theorem are **implied functions of a fourth variable**, that is, time. So we can find the derivative of the Pythagorean theorem with respect to time:

$$\frac{d}{dt}[a^2 + b^2 = c^2]$$

Using implicit differentiation:

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

Though we went from three variables to six, we know most of these… a is the distance of Car A to the intersection, that is 9 miles. b is 12 miles, and c is 15 miles. The new variables, namely the derivatives, represent the rate of change of each of the distances. In this case, $\frac{da}{dt}$ and $\frac{db}{dt}$ are given to us, and $\frac{dc}{dt}$ is what the question is asking for. So we can substitute:

$$2(9)(-35) + 2(12)(-25) = 2(15)\frac{dc}{dt}$$

And now we have a an equation with one unknown variable that we can solve. (Note, $\frac{da}{dt}$ and $\frac{db}{dt}$ are both negative here because both cars are getting closer to the intersection and therefore the distance is getting smaller). Finally,

$$\frac{dc}{dt} = -41 \text{ miles/hour}$$

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Another Example

Computer software is altering the size of a cylinder in graphic design software. The virtual radius of the cylinder is increasing by 1 meter per second and the virtual height of the cylinder is decreasing by 2 meters per second. What is the rate of change of the volume when the radius is 5 meters and the height is 11 meters.

First, start by writing the equation for volume of a cylinder:

$$V = \pi r^2 h$$

Then take the derivative with respect to time:

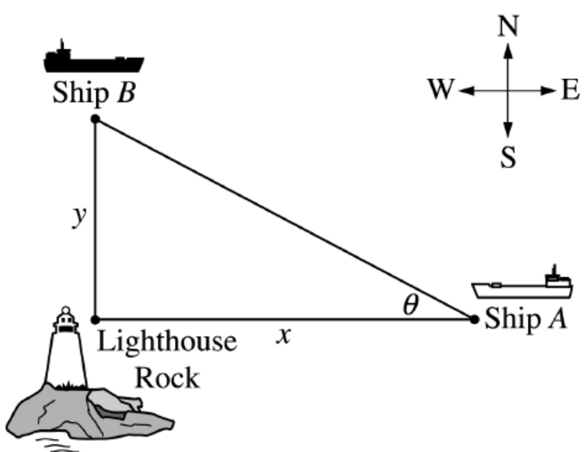
$$\frac{d}{dt}[V = \pi r^2 h]$$

$$\frac{dV}{dt} = 2\pi r h \frac{dr}{dt} + \pi r^2 \frac{dh}{dt}$$

Note that the derivative required the product rule as r^2 and h are both implicit functions of t. From here, we can substitute all known values and simplify:

$$\frac{dV}{dt} = 61\pi \text{ meters/second}$$

Practice (Ave Calculator)



Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t, as shown in the figure above.

(a) Find the distance, in kilometers, between Ship A and Ship B when $x = 4$ km and $y = 3$ km.

(b) Find the rate of change, in km/hr, of the distance between the two ships when $x = 4$ km and $y = 3$ km.

(c) Let θ be the angle shown in the figure. Find the rate of change of θ, in radians per hour, when $x = 4$ km and $y = 3$ km.

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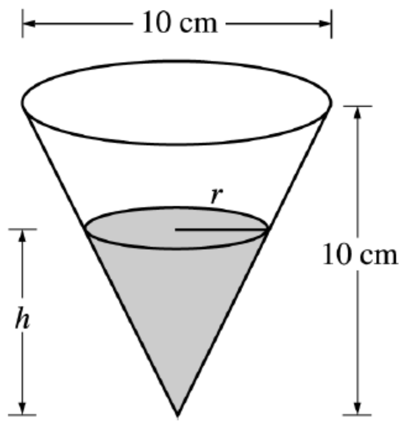
Practice (Sans Calcutron)



5. A container has the shape of an open right circular cone, as shown in the figure above. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $\frac{-3}{10}$ cm/hr.

   (Note: The volume of a cone of height h and radius r is given by $V = \frac{1}{3}\pi r^2 h$.)

   (a) Find the volume V of water in the container when $h = 5$ cm. Indicate units of measure.

   (b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate units of measure.

   (c) Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?